Chapter 6 FLUID DYNAMICS

Fluid dynamics:

The branch of physics in which we study the fluid when they are in motion is called fluid dynamics.

Dynamics→ Motion.

Fluid:

A substance which can flow or substance which is able to flow is called fluid.

- A fluid is a substance that deforms continuously under the application of a shear (tangential) stress no matter how small the shear stress may be.
- ⇒ The distinction between a fluid and the solid state of matter is clear if you compare fluid and solid behavior. A solid deforms when a shear stress applied but it does not continue to increase with time. However, if a shear stress is applied to a fluid, the deformation continues to increase as long as the stress is applied.
- ⇒ A fluid may be liquid or gas because they can flow.
  Solid can never flow so solid are not a fluid.

Flow: Continuous nature of motion of particles where we can't count them.

Layer: Path on which fluid particles move is known as layer. Or a fluid moves in the form of layers.

Some sub branches of fluid dynamics

- ⇒ **Hydro dynamics**: Liquid in motion and their properties.
- ⇒ **Hydro statics:** Liquid at rest and their properties.
- ⇒ **Aero dynamics:** Gases in motion and their properties.

Viscosity (Fluid friction):

- ⇒ The property of fluid due to which it opposes its flow. Or
- ⇒ Internal resistance or opposition between layers or particles of fluid which it opposes flow/motion of particles. Or
- ⇒ The resistance offered to the motion of fluid.
- ⇒ The effect of fluid friction or viscosity is to retard the motion of fluid.
- ⇒ The viscosity represents internal friction of fluid.
- ⇒ The greater the viscosity of fluid the greater is the force required to cause one layer of fluid to slide past another.
- ⇒ The viscosity of honey is very large as compared to that of water.
- ⇒ Note that viscosity of a fluid not only retards its own motion but it also retards motion of solid moving in it.
- ⇒ The greater viscosity of fluid the harder it is for a solid to move through it.
- ⇒ Now imagine how fast you would swim in water and that in honey. (ˇ – ˇ).
- ⇒ The force of attraction between the molecules of a moving fluid determine the viscosity of a fluid.
- ⇒ In order to maintain the liquid flow, an external force equal to drag force must be applied.
- ⇒ The viscosity of blood depends upon the concentration of red blood corpuscles; therefore, the viscosity of blood can be used to detect corpuscle deficiency.

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- ⇒ Viscosity of air or liquid is used in providing damping torque in measuring instrument.
- ⇒ Oil used as lubricants should have a proper value of viscosity
- ⇒ The viscosity of oil help in applying brakes.
- ⇒ Blood circulation through arteries depend upon the viscosity of blood.

Note: When solid surface moves over another solid surface their relative motion is opposed by the friction between the surface which is called solid friction.

Coefficient of viscosity:

Numerical value of viscosity of fluid is known as coefficient of viscosity.

$$F = \eta A \frac{dv}{dx}$$

η is a constant of proportionality and is called co − efficient of viscosity
its value depend upon nature of a fluid

- ➔ It is represented by eta (η).
- ➔ It is measure in a unit of kg/m-sec, Ns/m² (SI unit). It is also called Deca-poise or Pascal second (ps). The corresponding CGS unit is dyne-s/cm² called poise and is commonly used. 1 decapoise = 10 poise
- ➔ The viscosity of an ideal fluid is zero.
- ➔ Dimension: $[ML^{-1}T^{-1}]$

MCQS point:

$$\eta \propto \frac{1}{T} (liquid)$$
$$\eta \propto T (gases)$$

- ✓ With increase in temperature the electron of gases atom attains higher kinetic energy so the collision between electron increases with increase in collision cause viscosity.
- ✓ As thickness increased → viscosity increase
- ✓ Thickness decrease → viscosity decreases.

Fluidity (Flow rate):

- ⇒ The reciprocal of viscosity of fluid is called fluidity.
- ⇒ Represented by 'F'
- ⇒ Mathematically represented by: $F = {}^{1}/_{\eta}$
- ⇒ Flow rate are of two types
- ⇒ **i) Mass flow rate:** mass flow rate per unit time. $F_m = \frac{\Delta m}{\Delta t}$
- ⇒ **ii) Volume flow rate:** volume flow rate per unit time. $F_v = \frac{\Delta v}{\Delta t}$

Compressibility:

- ⇒ property of a fluid due to which we can change density of fluid by applying force or pressure and the fluid are known to be compressible fluid.
- ⇒ Gases are highly compressible
- ⇒ Liquids are slightly compressible

Types of flow

- ⇒ **Ideal flow:** if distance travelled by fluid particles is same in all layers of fluids.
- ⇒ **Real flow:** if distance travelled by fluid particles is not same in all layers.
- ⇒ **Streamline or laminar flow:** If particles of fluid move on parallel path, so that no particles cross any other layer.
- ⇒ **Turbulent flow:** If particles of different layers overlap.
- ⇒ **Rotational flow:** If fluid rotates about or around some fixed point.
- ⇒ **Viscous flow:** If energy is loss during flow.

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- **Non viscous flow:** If no energy is lost during flow.
- **Steady flow:** if all particles of fluid such as density, pressure and flow rate all are constant
- **Incompressible flow:** If density is constant during motion

Density of fluid:

In a fluid (which includes both liquid and gas), density ρ of a point mass is defined as "Mass per unit volume"

- $\rho = \frac{m}{v}$
- It is represented by rho (ρ).
- It is a scalar quantity.
- Units are kg/m^3 → SI, g/cc → C.G.S
- Relative density or specific gravity is defined as the the ratio of density of substance to density of water at 4°C. Hence,

$$R.D = \frac{Density\ of\ substance}{Density\ of\ water\ at\ 4degree\ celcius}$$

- **Density of a mixture of two or more liquids**

$$\rho = \frac{Total\ mass}{Total\ volume} = \frac{(m_1 + m_2)}{(v_1 + v_2)} = \frac{(m_1 + m_2)}{(\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2})}$$

$$if\ m_1 = m_2\ then\quad \rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2}$$

- **If two liquids of densities ρ_1 and ρ_2 having volumes V1 and V2 are mixed, then the density of mixture**

$$\rho = \frac{Total\ mass}{Total\ volume} = \frac{m_1 + m_2}{v_1 + v_2} = \frac{\rho_1 v_1 + \rho_2 v_2}{v_1 + v_2}$$

$$if\ v_1 = v_2 \rightarrow \rho = \frac{\rho_1 + \rho_2}{2}$$

- **Effect of temperature on density:** As the temperature of liquid is increased, the mass remains same while the volume is increased and hence, the density of liquid decreases (as $\rho = \frac{1}{v}$) thus

$$\frac{\rho'}{\rho} = \frac{v}{v'} = \frac{v}{v + dv} = \frac{v}{v + v_\varphi \theta}$$

$$\rho' = \frac{\rho}{1 + \varphi^\theta}$$

$$where\ \varphi = thermal\ co - efficient\ of\ volume\ expansion$$

$$\Delta\theta = rise\ in\ temperature.$$

- **Effect of pressure on density:** As pressure is increased, volume decreases and hence, density will decrease. Thus,

$$= \frac{\rho'}{\rho} = \frac{v}{v'} = \frac{v}{v + dv}$$

$$= \frac{v}{v - (\frac{dP}{B})v}$$

$$\frac{\rho'}{\rho} = \frac{1}{1 - \frac{dp}{B}} = \rho' = \frac{\rho}{1 - {dp}/{B}}$$

where dP = change in pressure
B = Bulk modulus of elasticity of liquid

Note: Drag force is the opposition to the body when it is moving in a fluid. On the other hand, the viscosity is an opposition between the layers of a fluid. No interference of another body.

Viscous Drag force

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- The force due to viscosity of fluid on a body moving in a fluid which opposes motion of a moving body is known as drag force.
- Or it may be defined as 'The force which opposes the motion of a body moving in a fluid.
- The magnitude of drag force for the first-time measure by stocks.

Stock's law:

- Stock's law gives us an equation of drag force.
- There is some **limitation** of this law → It is not valid for spherical object, → only valid for a body moving with small speed → Not valid for a body moving with high speed.
- Stocks law tells us that DRAG FORCE depend upon;
  - i) Viscosity
  - ii) Size
  - iii) Shape
  - iv) Orientation of body
  - v) Speed of body

Equation for drag force is given by

$$F_D \propto \eta$$
$$F_D \propto r$$
$$F_D \propto v (for\ small\ speed)$$
$$F_D \propto \eta r v$$
$$F_D = constant\ \eta r v$$
$$F_D = 6\pi\eta r v \quad (6\pi\ contsant\ of\ proportionality)$$

This equation is only valid for spherical object moving with slow speed.

- For any other shape and high speed, the drag force is equal to

$$F_D = \frac{1}{2}\rho c v^2 A$$

- Strictly the law applies to a fluid of infinite extent.
- The spherical body must be smooth and rigid.
- The stock's law is used in Millikan's (MILLIKAN OIL DROP) experiment for the determination of charge on electron.
- It accounts for the formation of clouds in sky.
- It explains why large rain drops hurt much more then small ones when they fall on you (your head especially when there is no hair), it is not just that they are heavier they are actually falling faster.
- It is used to find the size of small particles.
- It is useful in consideration of geological processes in which the rate of sedimentation is important.
- Stokes's law makes the following assumptions for the behavior of a particle in a fluid:

Laminar Flow
Spherical particles
Homogeneous (uniform in composition) material
Smooth surfaces
Particles do not interfere with each other.

- Note that for molecules Stokes's law is used to define their Stokes radius.
- The CGS unit of kinematic viscosity was named "stokes" after his work.

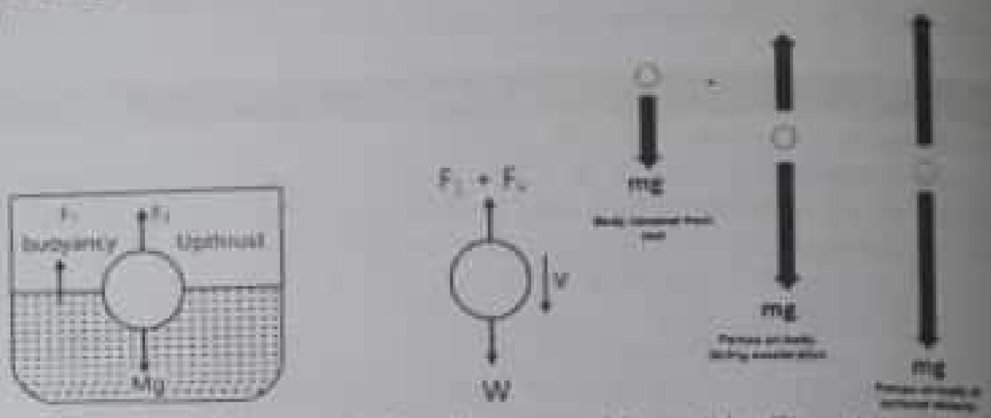
Terminal velocity (speed):

- It is the maximum constant speed gain by falling object in fluid when drag force becomes equal to the weight of a body is known as Terminal velocity.
- Terminal velocity → $F_D = W$.
- Consider a small sphere falls freely from rest through a large column of viscous fluid as shown in figure below then the forces acting on sphere would be:

I) Weight 'w' of sphere acting vertically downward.

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ii) Up-thrust 'U' equal to the weight of fluid displaced.
iii) Viscous drag force 'f' acting vertically upward in direction opposite to the motion of a body



⇒ Initially the downward force W is greater than the upward force (U + F) and the sphere accelerates downward. As the velocity of a sphere increases, the magnitude of velocity dependent viscous force (stock's law) also increases. Until a stage is reached when U + F = W since the resultant sphere on a sphere is zero. It moves downward with a constant maximum velocity called terminal velocity.

⇒ The expression for terminal velocity is given by,

$$v_t = \frac{2\rho g r^2}{9\eta}$$

Or

$$v_t = \sqrt{\frac{2mg}{\rho A C_d}} \text{ where } C_d = \text{drag coefficient}$$

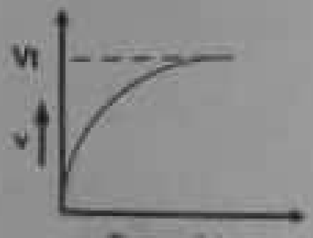
⇒ The expression of terminal velocity for spherical object falling through a fluid is,

$$v_T = \frac{2}{q} \times \frac{r^2(\rho - \sigma)g}{\eta}$$

⇒ Terminal velocity is directly proportional to mass of the object.
⇒ Terminal velocity is directly proportional to square of radius r^2 so if radius is made n - fold terminal velocity will become n2] times.
⇒ It shows that when light and heavier masses are allowed to fall simultaneously through the same fluid the lighter mass will attain terminal speed first where is the heavier mass will attain terminal speed later.
⇒ Large rain drops hurt much more it is also one of the reason that they have high terminal velocity
⇒ The terminal velocity of a spherical body is directly proportional to the difference in the densities of the body and fluid $(\rho - \sigma)$, if the density of a fluid is greater then that of body, $(\sigma > \rho)$. Then terminal velocity is negative. This means that body instead of falling moves upward. That's why bubbles of air rises up in air.
⇒ The terminal velocity of a spherical body is inversely proportional to the coefficient of viscosity of a fluid. It shows that more viscous the fluid, the smaller terminal velocity that the spherical body will acquire.
⇒ At moon the terminal velocity is 6 timeless that of earth. (let suppose the terminal velocity at earth is $6v_t$ then at moon the terminal velocity will be 1/6 v_t)
⇒ If η σ are known measuring terminal velocity provide information regarding. Size of a spherical particle. Millikan used the same method in determination of charge on electron (millikan oil drop method) → (Fsc part 1st, chemistry, chapter no 2, atomic structure).
⇒ Another factor that affects terminal velocity is the orientation at which a body falls.
⇒ $F_D \leq w$
⇒ $v \leq v_T$

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⇒ If a body has maximum drag force of (x N) if moving in fluid then its weight will also be (x). They will be same. if one is 40 N the weight will also be 40.
⇒ If $\rho > \sigma$ then terminal velocity will be positive and hence the spherical body will attain constant velocity in downward direction.
⇒ If $\rho < \sigma$ then terminal velocity will be negative and hence the spherical body will attain constant velocity in upward direction. Example: Air bubble in a liquid and clouds in sky.
⇒ Terminal velocity graph:



Terminal velocity of sky diver:

Fluid flow:

⇒ The study of fluid when they are in motion is called fluid dynamics.
⇒ The general flow of a fluid can be very complicated. let say: Consider the flow of a river in flood, rise of smoke from a burning cigarette.
⇒ While each drops of water in river or smoke particles is governed by newton second law. So, the resulting equation will be very complicated.
⇒ Therefore, we simplify the motion of fluid up to some specific extent due to which we are able to calculate many of properties of a particles of a fluid.
⇒ When we study the fluid, we follow some specific assumption they are;
- The fluid is non-viscous; i.e. There is no internal friction or resistance between the adjacent layers of fluid.
- The fluid is incompressible; i.e. The density of fluid is constant.
- The fluid motion is steady; (The steady flow of a fluid means that the velocity, density, and pressure of a fluid at each point in do not change with time.

⇒ With the help of these assumptions we can study the fluid in easy and precise way.

Types of liquid flow:

i) **Streamline flow or steady flow**
ii) **Turbulent flow**

1. Streamline flow:

⇒ If all liquid particles that pass any given follow the same path at the same speed (they have same velocity).
⇒ **Stream line:** The stream line is a curve whose tangent at any point is along the direction of velocity of the liquid particle at that point.
⇒ A bundle of streamlines is called tube of flow.
⇒ The streamlines represent path of particles.
⇒ In steady flow the velocity, density, and pressure at each point in the liquid don't change with time.
⇒ **Laminar flow:** It is a special case of streamline or steady flow in which the liquid flow as a series of parallel layer (laminar) and no one layer crosses another layer. The smooth streamline flow is known as laminar flow.

2. Turbulent flow:

⇒ The flow of a liquid is said to be turbulent or disorderly flow. If the speed and direction of liquid passing any point change with time.
⇒ Turbulent flow occurs in liquid when they are at high speed.

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⇒ When the velocity or speed of a liquid is small then flow is laminar or steady.
⇒ If the speed of fluid increases then it turns into turbulent flow.

Rate of flow:

⇒ The volume of a liquid that passes the cross section of a vessel, (i.e., pipe) in one second is called the rate of flow.
⇒ It is denoted by symbol 'Q'.
⇒ Rate of flow; $Q = \frac{v}{t}$
⇒ Its unit is m^3/s.
⇒ We know that: $V = a \times vt$
⇒ So, the equation becomes;

$$Q = \frac{v}{t}$$

$$so; \quad Q = \frac{a \times vt}{t}$$

$$Q = av$$

⇒ This is called discharge equation.
⇒ The equation shows that velocity of a fluid is inversely proportional to area of cross section of pipe.
⇒ If cross section area of pipe increases then the velocity of fluid decrease. And so on.

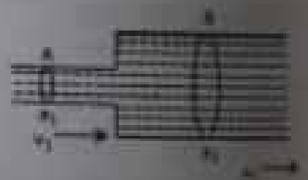
Reynolds number:

⇒ We know that when a liquid is flowing, then when they are at small speed their flow will be a stream line if their speed increases then a stage occur where the flow of this liquid become turbulent. Whether the flow will be laminar or turbulent is decide by Reynolds number (N_R)

$$Reynolds\ number = N_R = \frac{\rho v D}{\eta}$$

$$\rho = density\ of\ liquid$$
$$D = diameter\ of\ pipe$$
$$\eta = coefficient\ of\ viscosity\ of\ fluid$$
$$v = average\ velocity\ of\ flow$$

⇒ If N_R is less than 2000 the flow is laminar or steady.
⇒ If N_R is greater than 2000 the flow will be turbulent.
⇒ If N_R is in between 2000 and 3000 the flow is unstable it changes from laminar to turbulent and from turbulent again to laminar and so on.
⇒ N_R is pure number because all of its units cancel, and it determine nature of flow.
⇒ The value of N_R is independent of system of units used for measurement of various quantities.



Critical velocity:

⇒ When a fluid flowing in a tube has small speed there flow would be laminar.
⇒ When the speed of fluid that is flowing increased then there flow changes into turbulent.
⇒ That velocity of liquid flow up-to which its flow is steady and above which its flow changes into turbulent is called critical velocity.

$$critical\ velocity;\ v_c = \frac{1100\eta}{r\rho}$$

$$\eta = coefficient\ of\ viscosity$$
$$r = radius\ of\ pipe$$
$$\rho = density\ of\ pipe$$

Equation of continuity:

⇒ Hydro-dynamical equation which shows that mass flow rate of fluid in different points of pipe will be same or conserved.

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- This equation relates cross section area of pipe and speed of a fluid in that part of pipe.
- Equation of continuity is statement of 'conservation of mass'.
- In other words the equation of continuity states that 'the mass of liquid flowing through a pipe of varying cross section is constant when the density of a liquid does not change'. So it is a statement of conservation of mass.
- It states that for steady flow the mass flow into the volume must be equal to mass flow rate out.
- The equation of this principle is given by; $a_1v_1 = a_2v_2$

So

$$\left\{\frac{A_1}{A_2} = \frac{v_2}{v_1}\right\}$$

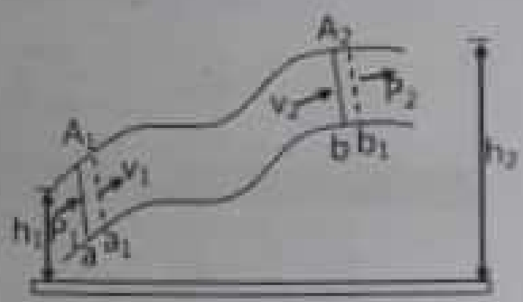
$$Av = \frac{\Delta v}{\Delta t} = constant$$

$$\frac{\Delta m}{\Delta t} = \rho\frac{\Delta V}{\Delta t}$$

$$\rightarrow A_1v_1 = A_2v_2 \rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2 \Rightarrow r_1^2 v_1 = r_2^2 v_2$$

$$\Rightarrow \left(\frac{d_1}{2}\right)^2 v_1 = \left(\frac{d_2}{2}\right)^2 v_2$$

$$d_1^2 v_1 = d_2^2 v_2$$



- When different quantities are given to find. Then you can easily calculate such as diameter, radius, velocity and area from above equation.
- These equations also show that the velocity of fluid is inversely proportional to area. $V \propto \frac{1}{A}$
- The deep part of the river has large cross section in comparison to shallow part of river, thus deep flows slowly in comparison to shallow water but the volume flow rates in both cases are same.

Bernoulli's theorem:

- It is simply a statement of 'law of conservation of energy'.
- It is represented by Swiss physicist Daniel Bernoulli's in 1738.
- This theorem relates the pressure of fluid to its speed and elevation.
- In deriving the Bernoulli's equation, we use the fluid that are: * Incompressible. * Non viscous. * Flow is non-turbulent; * and steady.
- The relation between pressure, flow speed and height for flow of an ideal fluid is called Bernoulli's theorem.
- The theorem states that "For the steady flow of an ideal liquid, the total energy (i.e. sum of pressure energy, potential energy, and kinetic energy) per unit volume remains constant throughout the flow.
- Mathematically we can write Bernoulli's theorem is;

$$p + \rho gh + \frac{1}{2}\rho v^2 = constant$$

$$p = pressure\ energy\ per\ unit\ voluume$$

$$\rho gh = potential\ energy\ per\ unit\ volume$$

$$\frac{1}{2}\rho v^2 = kinetic\ energy\ per\ unit\ volume$$

- The Bernoulli's theorem tells us that combination of evaluated three quantities at a point in the flow has same value as at any other point.
- The importance of this theorem lies in the fact that this theorem can be used to determine the variation of pressure in liquid as a function of velocity of the liquid and the elevation of the pipe through which the liquid is flowing.

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⇒ In derivation of Bernoulli's theorem, we use work energy principle.

⇒ Equation of Bernoulli's shows that for a liquid moving horizontally, if velocity v increases at a point, then pressure (p) decreases at that point and vice versa.

⇒ We can write the Bernoulli's equation in three different forms:

i) $p + \rho gh + \frac{1}{2}\rho v^2 = contstant$

In this equation all the three terms carry the dimension of pressure (ENERGY PER UNIT VOLUME)

ii) If we divide the above equation by Rho we get:

$$\frac{p}{\rho} + gh + \frac{1}{2}v^2 = constant$$

in this equation all the three terms carry the dimension of energy per unit mass.

iii) If we divide this second equation by g we get:

$$\frac{p}{\rho g} + h + \frac{v^2}{2g} = constant$$

Here all the three terms carry the dimenson of length.

Due to this reason this equation is called pressure head or velocity head.

Limitations of Bernoulli's equation:

i) In the derivation of this equation, it is assumed that the liquid is non-viscous, i.e. has zero viscosity or internal friction; however, a real fluid has some viscosity so that part of mechanical energy is lost to overcome liquid friction.

ii) In this equation, it is assumed that the rate of flow of liquid is constant ($a_1v_1 = a_2v_2$), but this is incorrect. In this case a liquid flowing through a pipe the velocity of flow is maximum at the center and goes on decreasing towards toward the wall of the pipe. Therefore, we should take average velocity.

iii) It is not assumed in this equation, that some of energy is lost. i.e... some of energy is change into heat and friction which was not taken in account.

Application of Bernoulli's theorem:

i) **Aero-foil lift:**



⇒ The are designed such like that air flows faster along the top surfaces that the lower one.

⇒ From Bernoulli's theorem the pressure of air below is greater than that of above.

⇒ It is the difference that provides a net upward force.

⇒ This is called dynamic lift.

⇒ Wings are usually tilted slightly upward so that air striking the bottom surface is directed downward.

ii) **Attraction between two closely parallel moving boats (or buses)**



When two boats or buses move side by side in the same direction, the water (or air) in the region between them moves faster than that on them remote sides. Consequently, in accordance with Bernoulli's principle the pressure between them is reduced and hence due to pressure difference they are pulled towards each other creating the so-called attraction.

iii) **Blowing off roofs by wind storms:**



⇒ During a tornado or hurricane, when a high-speed wind blows over a straw or tin roof, it creates a low pressure (P) in accordance with Bernoulli's principle.

⇒ However, the pressure below the roof (i.e., inside the room) is still atmospheric (= P_0). So due to this difference of pressure, the roof is lifted up and is then blown off by the wind.

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Jets and Nozzles:

iv)

⇒ According to Bernoulli's equation a slow stream of an ideal fluid can be converted into a narrowing the exit.

⇒ The smaller is the cross-section area, the greater is the increase in the speed and so greater is the pressure drop.



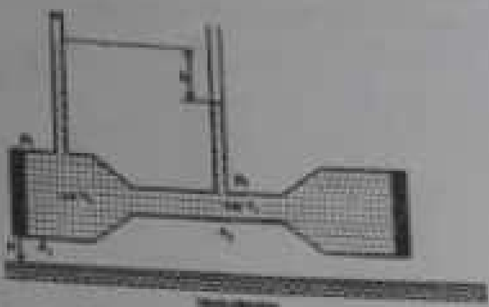
The action of carburetor, paint-gun, scent-spray or insect-sprayer is based on Bernoulli's principle. In all these, by means of motion of a piston P in a cylinder C, high speed air is passed over a tube T dipped in liquid L to be sprayed. High speed air creates low pressure over the tube due to which liquid (paint, scent, insecticide or petrol) rises in it and is then blown off in very small droplets with expelled air.

Flow meter or venture meter:

v)

⇒ A venturi meter (named after venture) is a device that is used to measure flow speed/ flow rate

⇒ It works on the basis of Bernoulli's theorem.



⇒ Mathematically flow rate can be find out by:

$$v_1 = \sqrt{\frac{2gh}{\left(\frac{A_1^2}{A_2^2}-1\right)}}$$

⇒ Knowing A_1, A_2 and h v_1 can be found and hence the rate of flow of liquid A_1v_1 can be calculated.

So

$$v \propto \frac{1}{A} \Rightarrow p \propto \frac{1}{v}$$

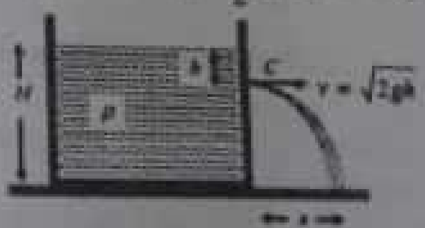
Also, it can be written as;

$$Q = A_1v_1 = A_1A_2 = \sqrt{\frac{2gh}{A_1^2 - A_2^2}}$$

Torricelli's theorem (SPEED OF EFFLUX):

⇒ If a liquid is filled in a vessel up to height H and a hole is made at a depth h below the free surface of the liquid as shown in fig. then taking the level of hole as reference level (i.e., zero point of potential energy) and applying Bernoulli's principle to the liquid just inside and outside the hole (assuming the liquid to be at rest inside) we get

$$(p_0 + h\rho g) + 0 = p_0 + \frac{1}{2}\rho v^2 \text{ or } V = \sqrt{2gh}$$



Which is same as the speed that an object would acquire in falling from rest through a distance h and is called velocity of efflux or velocity of flow?

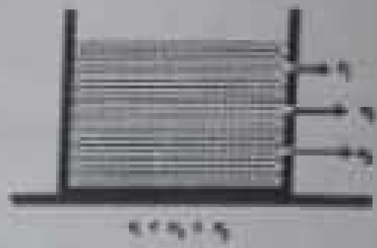
This result was first given by Torricelli, so this is known as Torricelli's theorem.

⇒ The velocity of efflux is independent of the nature of liquid, quantity of liquid in the vessel and the area of orifice.

⇒ Greater is the distance of the hole from the free surface of liquid, greater will be the velocity of efflux

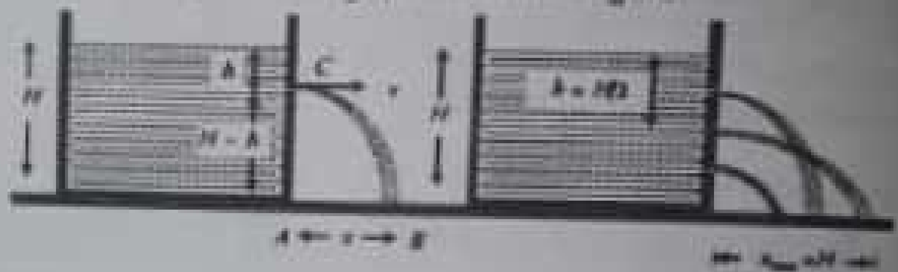
i.e. $v \propto \sqrt{h}$

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⇒ As the vertical velocity of liquid at the orifice is zero and it is at a height (H - h) from the base the time taken by the liquid to reach the base-level $t = \sqrt{\frac{2(H-h)}{g}}$

⇒ Now during time t liquid is moving horizontally with constant velocity v, so it will hit the base level at a horizontal distance x (called range) as shown in figure



Such that

$$x = vt = \sqrt{2gh} \times \sqrt{[2(H-h)/g]} = 2\sqrt{h(H-h)}$$

For maximum range.

$$\frac{dx}{dh} = 0$$

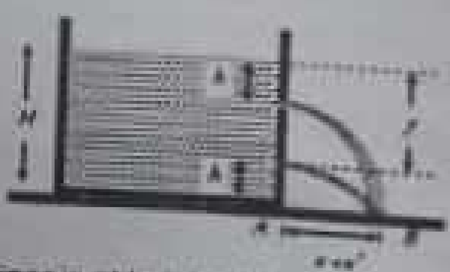
$$h = \frac{H}{2}$$

i.e., range x will be maximum when

$$h = \frac{H}{2}$$

$$Maximum\ range \Rightarrow x_{max} = 2\sqrt{\frac{H}{2}\left[H - \frac{H}{2}\right]} = H$$

⇒

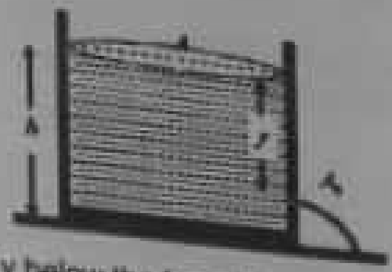


- If the level of free surface in a container is at height H from the base and there are two holes at depth h and y below the free surface, then

$$x = 2\sqrt{h(H-h)}\ and\ x' = 2\sqrt{y(H-y)}$$
$$Now\ if\ x = x'\ i.e\ h(H-h) = y(H-y)$$
$$i.e.,\ y^2 - Hy + h(H-h) = 0$$
$$or,\ y = \frac{1}{2}[H \pm (H-2h)]$$
$$i.e,\ y = h\ or\ (H-h)$$

i.e., the range will be same if the orifice is at a depth h or $(H-h)$ below the free surface. Now as the distance $(H-h)$ from top means $H-(H-h) = h$ from the bottom, so the range is same for liquid coming out of holes at same distance below the top and above the bottom.

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If A_0 is the area of orifice at a depth y below the free surface and A is that of container, the volume of liquid coming out of the orifice per second will be

$$\left(\frac{dv}{dt}\right) = VA_0 = A_0\sqrt{2gy} \qquad (As\ v = \sqrt{2gy})$$

Due to this, the level of liquid in the container will decrease and so if the level of liquid in the container above the hole changes from y to y − dy in time t to t + dt then −dV = A dy

So substituting this value of dV in the above equation

$$-A\frac{dy}{dt} = A_0\sqrt{2gy}$$

$$i.e., \int dt = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int y^{-1/2}dy$$

So the time taken for the level to fall from H to H'

$$t = -\frac{A}{A_0}\frac{1}{\sqrt{2g}}\int_H^{H'} y^{-1/2}dy = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right]$$

If the hole is at the bottom of the tank, time t to make the tank empty

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}} \qquad (As\ here\ H' = 0)$$

vi) Entrainment

⇨ People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called entrainment. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in Figure.



⇨ Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another.
(a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids.
(c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel.
(d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

vii) Power in Fluid Flow:

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- Power is the rate at which work is done or energy in any form is used or supplied. To see the relationship of power to fluid flow, consider Bernoulli's equation:

$$P + \frac{1}{2}\rho v^2 + \rho g h = constant$$

All three terms have units of energy per unit volume, as discussed in the previous section. Now, considering units, if we multiply energy per unit volume by flow rate (volume per unit time), we get units of power. That is, (E/V)(V/t)=E/t. This means that if we multiply Bernoulli's equation by flow rate Q, we get power. In equation form, this is

$$(P + \frac{1}{2}\rho v^2 + \rho g h)Q = power$$

Each term has a clear physical meaning. For example, PQ is the power supplied to a fluid, perhaps by a pump, to give it its pressure P. Similarly, ½ ρv² Q is the power supplied to a fluid to give it its kinetic energy. And $\rho g h\, Q$ is the power going to gravitational potential energy.

Formula Sheet

S/N	QUANTITY	EQUATION
1)	*fluidity*	$F = \frac{1}{\eta}, \eta \rightarrow viscosity$
2)	*Mass flow rate*	$F_m = \Delta m/\Delta t$
3)	*Volume flow rate*	$F_v = \Delta v/\Delta t$
4)	*Stock's Law for spherical object and low speed*	$F_D = 6\pi\eta r v$
5)	*Stock's law for any shape and high speed*	$F_D = \frac{1}{2}\rho c v^2 A$
6)	*Terminal velocity*	$V_t = \frac{mg}{6\pi\eta r v}$ For spherical object: $V_t = \frac{2\rho r^2}{9\eta}$
7)	*Continuity equation*	$A_1V_1 = A_2V_2$ $\frac{A_1}{A_2} = \frac{v_2}{v_1}$ $r_1^2 v_1 = r_2^2 v_2$ $v_1 D_1^2 = v_2 D_2^2, D \rightarrow Diameter$
8)	*Bernollie's Equation*	$P_1 + \rho g h + \frac{1}{2}\rho v_1^2 = P_2 + \rho g h_2 + \frac{1}{2}\rho v_2^2$ $\frac{K.E}{V} = \frac{1}{2}\rho v^2$ $\frac{P.E}{v} = \rho g h$ $\Delta P = \rho g(h_2 - h_1) \rightarrow pascal$ $\Delta P = \frac{1}{2}\rho(v_2^2 = v_1^2)$ $F = \frac{1}{2}\rho A(v_2^2 - v_1^2)$
9)	*Speed of efflux*	$V_{ef} = \sqrt{2gh}$

Some Important Points and Tricks:

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1) When a liquid is in equilibrium, the force acting on its surface is perpendicular everywhere.
2) In a liquid, the pressure is same at the same horizontal level.
3) Pressure at any point is same in all directions.
4) The pressure is perpendicular to the surface of the fluid.
5) The pressure at any point in the liquid depends on depth (h) below the surface, density of liquid and acceleration due to gravity.
6) It is independent of the shape of the containing vessel, or total mass of the liquid.
7) Force is a vector quantity but pressure is a tensor quantity.
8) Pressure and density play the same role in case of fluids as force and mass play in case of solids.
9) Bar and mill bar are commonly used units for pressure in meteorology.
10) Sudden fall in atmospheric pressure predicts possibility of a storm.
11) Water barometer was constructed in 17th century by Von Guericke and fixed on the outside wall of his house. With the help of this barometer Von Guericke made the first recorded scientific weather forecast. He correctly predicted the severe storm after noting a sudden fall in the height of the water column.
12) The specific gravity is also known as rel ative density. Thus, S.G. of a Substance = density of the substance/densityofwater (at 4 °C)
13) If the specific gravity of the material of a body is x, then its density is
    (I) x g cm^{-3} in C.G.S.
    (II) x × 10^3 kg m^{-3} in SI.
14) The number of moles in a sample of any substance containing N molecules is given by

$$\mu = \frac{N}{N_A}$$

15) The force between atoms and molecules is electrical in nature. However, it does not obey inverse square law.
16) If two liquids of masses m_1, m_2 and densities ρ_1, ρ_2 are mixed together, then the density of the mixture is given by

$$\rho = \frac{m_1 + m_2}{\frac{m_1}{\rho_1} + \frac{m_2}{\rho_2}}$$

17) If two liquids of same mass but different densities are mixed together, then the density of the mixture is harmonic mean of the densities. That is

$$\rho = \frac{2\rho_1\rho_2}{\rho_1 + \rho_2} \text{ or } \frac{1}{\rho} = \frac{1}{2}\left[\frac{1}{\rho_1} + \frac{1}{\rho_2}\right]$$

18) It two drops of same volume but different densities are mixed together, then the density of the mixture is arithmetic mean of the densities, that is

$$\rho = \frac{\rho_1 + \rho_2}{2}$$

19) The density of the liquid changes with pressure as follows:

$$\rho = \rho_0\left[1 + \frac{\Delta p}{B}\right]$$

Where;
Δp = change in pressure and B is the bulk moduli

20) The density of a liquid of bulk modules B at depth h is given

$$\rho_d = \rho_0\left[\frac{1 + h\rho g}{B}\right]$$

Where ρ is the average density of liquid.

21) The hydrometer can be used to measure density of the liquid or fluid. If a vessel contains liquid up to a height H and it has a hole in the side at a height h, then the velocity of efflux is $v = \sqrt{2g(H-h)}$. The time taken by the liquid to reach the ground level is $t = \sqrt{2h/g}$. Horizontal range of the liquid $R = 2[h(H-h)]^{1/2}$. The range is same for the hole at a height h above the bottom or at the depth h below the surface of the liquid.
22) The cross-section of the water stream from a tap decreases as it goes down in accordance with the equation of continuity.
23) The up thrust on body immersed in a liquid does not depend on the mass, density or shape of the body. It only depends on the volume of the body.

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24) The weight of the plastic bag full of air is same as that of the empty bag because the up thrust is equal to the weight of the air enclosed.

25) Up thrust depends on the density of the fluid, not the density of the body.

26) If two bodies have equal up thrust in a liquid, both have the same volume.

27) When air blows over a roof, the force on the roof is upwards.

28) If one floats one's back on the surface of water, the apparent weight is zero.

29) If a body just floats in liquid (density of the body is equal to the density of liquid) then the body sinks if it is pushed downwards.

30) The line joining the Centre of gravity and Centre of buoyancy is called central line.

31) The point where the vertical line through Centre of buoyancy intersects the central line is called metacenter.

32) The floating body is in stable equilibrium where the metacenter is above the Centre of gravity. (Centre of gravity is below the Centre of buoyancy)

33) The floating body is in unstable equilibrium when the metacenter lies below the Centre of gravity. (Centre of gravity is above the Centre of buoyancy)

34) The floating body is in the neutral equilibrium when Centre of gravity coincides with the metacenter. (Centre of gravity coincides with the Centre of buoyancy)

35) The wooden rod cannot float vertically in a pond of water because Centre of gravity lies above the metacenter.

36) Air bubble in water always goes up. It is because density of air (ρ) is less than the density of water (σ). So the terminal velocity for air bubble is negative, which implies that the air bubble will go up. Positive terminal velocity means the body will fall down.

37) The faster the air, the lower the pressure.

38) Wings of an aero0plane are shaped to make air travel further and faster over their top surfaces.

39) The lift force on a wing or aero-foil is proportional to the square of the speed of flow.

40) Viscous force between the layers of a liquid is analogous to friction between two solid surfaces.

41) With increase in temperature, the coefficient of viscosity of liquids decreases but that of gases increases. The reason is that as temperature rises the atoms of the liquid become more mobile, whereas in case of a gas, the collision frequency of atoms increases as their motion becomes more random.

42) We cannot sip a drink with a straw on the moon because there is no atmosphere on the moon.

MULTIPLE CHOICE QUESTIONS

1) Equal masses of water and a liquid of density 2 are mixed together, then the mixture has a density of
(a) 2/3 (b) 4/3 (c) 3/2 (d) 3

2) The pressure at the bottom of a tank containing a liquid does not depend on?
(a) Acceleration due to gravity
(b) Height of the liquid column
(c) Area of the bottom surface
(d) Nature of the liquid

3) An ice berg of density 900 Kg/m3 is floating in water of density 1000 Kg/m3. The percentage of volume of ice-cube outside the water is
(a) 20% (b) 35% (c) 10% (d) 25%

4) In which one of the following cases will the liquid flow in a pipe be most streamlined
(a) Liquid of high viscosity and high density flowing through a pipe of small radius
(b) Liquid of high viscosity and low density flowing through a pipe of small radius
(c) Liquid of low viscosity and low density flowing through a pipe of large radius
(d) Liquid of low viscosity and high

density flowing through a pipe of large radius

5) Two water pipes of diameters 2 cm and 4 cm are connected with the main supply line. The velocity of flow of water in the pipe of 2 cm diameter is
(a) 4 times that in the other pipe
(b) $\frac{1}{4}$ times that in the other pipe
(c) 2 times that in the other pipe
(d) $\frac{1}{2}$ times that in the other pipe

6) Water enters through end A with speed v_1 and leaves through end B with speed v_2 of a cylindrical tube AB. The tube is always completely filled with water. In case I tube is horizontal and in caseII it is vertical with end A upwards and in case III it is vertical with end B upwards. We have $v_1 = v_2$ for
(a) Case I (b) Case II
(c) Case III (d) Each case

7) Water is moving with a speed of 5.18 ms^{-1} through a pipe with a cross-sectional area of 4.20 cm^2. The water gradually descends 9.66 m as the pipe increase in area to 7.60 cm^2. The speed of flow at the lower level is
(a) 3.0 ms^{-1} (b) 5.7 ms^{-1}
(c) 3.82 ms^{-1} (d) 2.86 ms^{-1}

8) The velocity of kerosene oil in a horizontal pipe is 5 m/s. If g = 10m /s^2 then the velocity head of oil will be
(a) 1.25 m (b) 12.5 m
(c) 0.125 m (d) 125 m

9) A manometer connected to a closed tap reads 3.5×10^5 N/m^2. When the valve is opened, the reading of manometer falls to 3.0×10^5 N/m^2, then velocity of flow of water is
(a) 100 m/s (b) 10 m/s
(c) 1 m/s (d) $10\sqrt{10}\frac{m}{s}$

10) Air is streaming past a horizontal air plane wing such that its speed in 120 m/s over the upper surface and 90 m/s at the lower surface. If the density of air is 1.3 kg per metre3 and the wing is 10 m long and has an average width of 2 m, then the difference of the pressure on the two sides of the wing of
(a) 4095.0 Pascal (b) 409.50 Pascal
(c) 40.950 Pascal (d) 4.0950 Pascal

11) A large tank filled with water to a height 'h' is to be emptied through a small hole at the bottom. The ratio of time taken for the level of water to fall from h to $\frac{h}{4}$ and from $\frac{h}{2}$ to zero is ____________?
(a) $\sqrt{2}$ (b) $\frac{1}{\sqrt{2}}$ (c) $\sqrt{2}-1$ (d) $\frac{1}{\sqrt{2}-1}$

12) A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in m/s) through a small hole on the side wall of the cylinder near its bottom is
(a) 10 (b) 20 (c) 25.5 (d) 5

13) There is a hole in the bottom of tank having water. If total pressure at bottom is 3 atm (1 atm = 105N/m^2) then the velocity of water flowing from hole is
(a) $\sqrt{400}\frac{m}{s}$ (b) $\sqrt{600}\frac{m}{s}$
(c) $\sqrt{60}\frac{m}{s}$ (d) *None*

14) There is a hole of area A at the bottom of cylindrical vessel. Water is filled up to a height h and water flows out in t second. If water is filled to a height 4h, it will flow out in time equal to
(a) t (b) 4t (c) 2 t (d) t/4

15) A cylindrical tank has a hole of 1 cm^2 in its bottom. If the water is allowed to flow into the tank from a tube above it at the rate of 70 cm^3/sec. then the maximum height up to which water can rise in the tank is
(a)2.5 cm (b)5 cm (c)10 cm (d) 0.25 cm

16) A square plate of 0.1 m side moves parallel to a second plate with a velocity of 0.1 m/s, both plates being immersed in water. If the viscous force is 0.002 N and the coefficient of viscosity is 0.01 poise, distance between the plates in m is
(a) 0.1 (b) 0.05
(c) 0.005 (d) 0.0005

17) Spherical balls of radius 'r' are falling in a viscous fluid of viscosity 'η' with a velocity 'V'. The retarding viscous force acting on the spherical ball is
(a) Inversely proportional to 'r' but directly proportional to velocity 'V'
(b) Directly proportional to both radius 'r' and velocity 'V'
(c) Inversely proportional to both radius 'r' and velocity 'V'

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(d) Directly proportional to 'r' but inversely proportional to 'v'

18) Two drops of the same radius are falling through air with a steady velocity of 5 cm per sec. If the two drops coalesce, the terminal velocity would be
(a) 10 cm per sec (b) 2.5 cm per sec
(c) $5 \times (4)^{\frac{1}{3}} \frac{cm}{s}$ (d) $5 \times \sqrt{2} cm/s$

19) In a laminar flow the velocity of the liquid in contact with the walls of the tube is
(a) Zero (b) Maximum
(c) In between zero and maximum
(d) Equal to critical velocity

20) In a turbulent flow, the velocity of the liquid molecules in contact with the walls of the tube is
(a) Zero (b) Maximum
(c) Equal to critical velocity
(d) May have any value

21) Water is flowing through a tube of non-uniform cross-section ratio of the radius at entry and exit end of the pipe is 3:2. Then the ratio of velocities at entry and exit of liquid is
(a) 4:9 (b) 9:4 (c) 8:27 (d) 1:1

22) Water is flowing through a horizontal pipe of non-uniform cross-section. At the extreme narrow portion of the pipe, the water will have
(a) Maximum speed and least pressure
(b) Maximum pressure and least speed
(c) Both pressure and speed maximum
(d) Both pressure and speed least

23) In a streamline flow
(a) The speed of a particle always remains same
(b) The velocity of a particle always remains same
(c) The kinetic energies of all the particles arriving at a given point are the same
(d) The moments of all the particles arriving at a given point are the same

24) An application of Bernoulli's equation for fluid flow is found in
(a) Dynamic lift of an aero plane
(b) Viscosity meter
(c) Capillary rise (d) Hydraulic press

25) The Working of an atomizer depends upon
(a) Bernoulli's theorem (b) Boyle's la...
(c) Archimedes principle
d) Newton's law of motion

26) According to Bernoulli's equation

$$\frac{P}{\rho g} + h + \frac{1}{2}\frac{v^2}{g} = \text{constant}$$

The terms A, B and C are generally called respectively
(a) Gravitational head, pressure head and velocity head
(b) Gravity, gravitational head and velocity head
(c) Pressure head, gravitational head and velocity head
(d) Gravity, pressure and velocity head

27) The pans of a physical balance are in equilibrium. Air is blown under the right hand pan; then the right hand pan will
(a) Move up (b) Move down
(c) Move erratically (d) Remain at the same level

28) At what speed the velocity head of a stream of water be equal to 40 cm of Hg
(a) 282.8 cm/sec (b) 432.6 cm/sec
(c) 632.6 cm/sec (d) 832.6 cm/sec

29) The weight of an aero plane flying in air is balanced by
(a) Up thrust of the air which will be equal to the weight of the air having the same volume as the plane
(b) Force due to the pressure difference between the upper and lower surfaces of the wings, created by different air speeds on the surface
(c) Vertical component of the thrust created by air currents striking the lower surface of the wings
(d) Force due to the reaction of gases ejected by the revolving propeller

30) In this figure, an ideal liquid flows through the tube, which is of uniform cross-section. The liquid has velocities v_A and v_B, and pressure P_A and P_B at points A and B respectively
(a) $v_A = v_B$ (b) $v_B > v_A$
(c) $P_A > P_B$ (d) $P_B > P_A$

31) A sniper fires a rifle bullet into a gasoline tank making a hole 53.0 m below the surface of gasoline. The tank was sealed at 3.10 atm. The stored gasoline has a density of 660 kgm^{-3}. The velocity

with which gasoline begins to shoot out of the hole is

(a) 27.8 ms^{-1} (b) 41.0 ms^{-1}

(c) 9.6 ms^{-1} (d) 19.7 ms^{-1}

32) A cylindrical vessel of [illegible] cm height is kept filled up-to the brim. It has four holes 1, 2, 3, 4 which are respectively at heights of 20 cm, 30 cm, 45 cm and 50 cm from the horizontal floor. The water falling at the maximum horizontal distance from the vessel comes from

(a) Hole number 4 (b) Hole number 3

(c) Hole number 2 (d) Hole number 1

33) A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to $\frac{H}{\eta}$ (η < 1), and it takes T_2 time to take out the rest of water. If $T_1 = T_2$ then value of η will be______? (a) 2 (b) 3 (c) 4 (d) $2\sqrt{2}$

34) Velocity of water in a river is

(a) Same everywhere

(b) More in the middle and less near its banks

(c) Less in the middle and more near its banks

(d) Increase from one bank to other bank

35) As the temperature of water increases, its viscosity

(a) Remains unchanged (b) Decreases

(c) Increases

(d) Increases or decreases depending on the external pressure

36) The coefficient of viscosity for hot air is

(a) Greater than the coefficient of viscosity for cold air

(b) Smaller than the coefficient of viscosity for cold air

(c) Same as the coefficient of viscosity for cold air

(d) Increases or decreases depending on the external pressure

37) A good lubricant should have

(a) High viscosity (b) Low viscosity

(c) Moderate viscosity (d) High density

38) We have three beakers A, B and C containing glycerin, water and kerosene respectively. They are stirred vigorously and placed on a table. The liquid which comes to rest at the earliest is

(a) Glycerin (b) Water

(c) Kerosene (d) All of them at the same time

39) A small drop of water falls from rest through a large height h in air, the final velocity is

(a) $\propto \sqrt{h}$ (b) $\propto h$

(c) $\propto \left(\frac{1}{h}\right)$ (d) almost independent of h

40) Water is flowing in a pipe of diameter 4 cm with a velocity 3 m/s. The water then enters into a tube of diameter 2 cm. The velocity of water in the other pipe is

(a) 3 m/s (b) 6 m/s

(c) 12 m/s (d) 8 m/s

41) Water falls from a tap, down the streamline

(a) Area decreases (b) Area increases

(c) Velocity remains same

(d) Area remains same

42) Viscous forces are not present in

a. rotational flow b. irrotational flow

c. laminar flow d. none of the

43) If viscosity of fluid is more, the thickness of boundary layer is

a. more b. less c. not affected by change in viscosity

d. unpredictable

44) The radius of small fog droplet in air is found to be 5.1 x 10^{-6} m. the co-efficient of viscosity of air is 1.9 x 10^{-5} kg m^{-1} s^{-1}. Then what will be stiling speed of the droplet in air______________?

(a) 2.98 x 10^{-3} ms^{-1} (b) 1.5 x 10^{-6} ms^{-1}

(c) 44.6 x 10^{11} ms^{-1} (d) 0 ms^{-1}

45) Velocity of water in 6 inch diameter ipe is 5ft s-1. Find thhe velocity in a 3 inch diameter pipe which connect with it________? [both pipes flow full]

(a) 60 ft/s (b) 30 ft/s

(c) 40 ft/s (d) 20 ft/s

46) A water hose of internal diameter 20 m at the outlet discharges 30 kg of water in 60 s. then what will be the water speed at the outlet. Flow is steady and density of water is equal to 1000 kg m^{-3}?

(a) 256 ms^{-1} (b) 345 ms^{-1}

(c) 123 ms^{-1} (d) 1592.3 ms^{-1}

47) Bernoulli's equation is an accordance with____________?

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(a) Law of conservation of mass
(b) Law of conservation of energy
(c) Law of conservation of power
(d) Law of conservation volume

48) Bernoulli's equation relates which of the following quantities_____________?
(a) Pressure of fluid with speed of fluid
(b) Pressure to volume (c) Pressure to elevation (d) Both a and c is correct

49) the unit of ρgh is same as that of?
(a) Volume (b) Area
(c) Pressure (d)Velocity

50) the equation: $p + \frac{1}{2}\rho v^2 + \rho gh =$ constant implies that_____________?
(a)velocity of liquid is greater pressure will be greater
(b)when velocity of liquid is less pressure will be high and vice versa
(c)when velocity of liquid is less pressure will be less
(d) all are correct

51) 2m high tank is full of water a hollow appears at its middle, speed of efflux will be__________?
(a) 22.34 m/s (b) 456 m/s
(c)4.42 m/s (d) 0 m/s

52) A rain drop of radius "r" fall in air with a terminal speed v_t what is the terminal speed of radius 2r__________?
(a) $V_t/2$ (b) V_t
(c) $4v_t$ (d) $2v_t$

53) Eight spherical rain drops of same mass and radius are falling down with a terminal velocity of $6cms^{-1}$ if they collapse to form 1 big drop what will be its terminal speed__________? [neglect the buoyant force or buoyancy due to air]
(a) 1.5 cms^{-1} (b) 6 cms^{-1}
(c) 24 cms^{-1} (d) 32 cms^{-1}

54) According to equation of continuity when water falls its speed increases while its cross sectional area(a) increases (b) decreases (c) remain same (d) different

55) Venturi relation is one of applications of (a) equation of continuity (b) Bernoulli's equation (c) light equation (d) speed equation

56) the terminal velocity of a spherical ball of lead of radius r falling through a viscous liquid varies with radius r such that_______?
A: v_t/r = constant B: $v_t r$ = constant
C: $v_t r$ = constant D: v_t/r^2 = constant

57) after terminal velocity is reached the acceleration of a body falling through a fluid is________?
A: Equal to g B: Zero
C: Less than g D: More than g

58) the difference in pressure between the lower and upper surface of air plane___?
A. Depend upon attitude of plane
B. Independent of attitude
C. Insufficient data
D. None of these

59) two hail stones of radii in the ratio of 1:2 fall from a great hight through the atmosphere then there terminal velocity are in the ratio of________?
A 1:2 B: 4:1 C: 1 : 4 D:2:1

60) An incompressible fluid that flows through a cylindrical pipe of radius 2r at point A having radius r at point B further along the direction of flow if the velocity at point A is v then velocity at point B is________?
A: 2v B: V C: v/2 D: 4v

ANSWERS WITH EXPLANATION

1) (b) If two liquid of equal masses and different densities are mixed together then density of mixture

$$\rho = \frac{2\rho_1\rho_2}{\rho_1+\rho_2} = \frac{2\times1\times2}{1+2} = \frac{4}{3}$$

2) (c) $P = h\rho g$ i.e. pressure does not depend upon the area of bottom surface.

3) (c) Let the total volume of ice-berg is V and its density is ρ. If this ice-berg floats in water with volume V_{in} inside it then

$$V_{in}\sigma g = V\rho g \Rightarrow V_{in} = (\frac{\rho}{\sigma})V$$

$$\text{or } V_{out} = V - V_{in} = (\frac{\sigma-\rho}{\sigma})V$$

$$\Rightarrow \frac{V_{out}}{V} = (\frac{\sigma-\rho}{\sigma}) = \frac{1000-900}{1000} = \frac{1}{10}$$

$$\therefore V_{out} = 10\% \text{ of } V$$

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4) (b) For streamline flow, Reynold's number $N_R \propto \frac{\rho v}{\eta}$ hould be less. For less value of N_R, radius and density should be small and viscosity should be high.

5) (a) $d_A = 2\,cm$ and $d_B = 4\,cm \therefore r_A = 1\,cm$ and $r_B = 2\,cm$

From equation of continuity, av = constant

$$\frac{v_A}{v_B} = \frac{a_B}{a_A} = \frac{\pi(r_B)^2}{\pi(r_A)^2} = \left(\frac{2}{1}\right)^2 \Rightarrow v_A = 4v_B$$

6) (d) This happens in accordance with equation of continuity and this equation was derived on the principle of conservation of mass and it is true in every case, either tube remain horizontal or vertical.

7) (d)

$$A_1v_1 = A_2v_2 \Rightarrow 4.20 \times 5.18 = 7.60 \times v_2 \Rightarrow v_2 = 2.86\,m/s$$

8) (a) Velocity head $h = \frac{v^2}{2g} = \frac{(5)^2}{2\times10} = 1.25\,m$

9) (b) Bernoulli's theorem for unit mass of liquid

$$\frac{P}{\rho} + \frac{1}{2}v^2 = constant$$

As the liquid starts flowing, it pressure energy decreases

$$\frac{1}{2}v^2 = \frac{P_1 - P_2}{\rho} \Rightarrow \frac{1}{2}v^2 = \frac{3.5\times10^5 - 3.0\times10^5}{10^3} = v^2$$

$$\therefore \frac{2\times0.5\times10^5}{10^3} \Rightarrow v^2 = 100 \Rightarrow v = 10\,m/s$$

10) (a) From the Bernoulli's theorem

$$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$$= \frac{1}{2}\times1\times[(120)^2 - (90)^2] = 4095.0\frac{N}{m^2} \text{ or pascal}$$

Note $\frac{N}{m^2} = Pascal$

11) (c) Time taken for the level to fall from H to H'

$t = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{H} - \sqrt{H'}\right]$ ⇒According to problem- the time taken for the level to fall from h to $\frac{h}{2}$ $t_1 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{h} - \sqrt{\frac{h}{2}}\right]$

and similarly time taken for the level to fall from $\frac{h}{2}$ to zero $t_2 = \frac{A}{A_0}\sqrt{\frac{2}{g}}\left[\sqrt{\frac{h}{2}} - 0\right]$

$$\therefore \frac{t_1}{t_2} = \frac{1 - \frac{1}{\sqrt{2}}}{\frac{1}{\sqrt{2}} - 0} = \sqrt{2} - 1$$

12) (b)

$$v = \sqrt{2gh} = \sqrt{2\times10\times20} = 20\,m/s$$

13) (a) Pressure at the bottom of tank $P = \rho gh = 3\times10^5\frac{N}{m^2}$

Pressure due to liquid column

$$P_1 = 3\times10^5 - 1\times10^5 = 2\times10^5$$

and velocity of water $v = \sqrt{2gh}$

$$v = \sqrt{\frac{2P_1}{\rho}} = \sqrt{\frac{2\times2\times10^5}{10^3}} = \sqrt{400}\,m/s$$

14) (c) Time required to emptied the tank

$$t = \frac{A}{A_0}\sqrt{\frac{2H}{g}} \therefore \frac{t_2}{t_1} = \sqrt{\frac{H_2}{H_1}} = \sqrt{\frac{4H}{h}} = 2 \therefore t_2 = 2t$$

15) (a) The height of water in the tank becomes maximum when the volume of water flowing into the tank per second becomes equal to the volume flowing out per second

Volume of water flowing out per second

$$= Av = A\sqrt{2gh} \quad ...(i)$$

Volume of water flowing in per second = $70\frac{cm^3}{sec}$ ...(ii)

From (i) and (ii) we get

$$A\sqrt{2gh} = 70 \Rightarrow 1\times\sqrt{2gh} = 70$$

$$\Rightarrow 1\times\sqrt{2\times980\times h} = 70 \therefore h = \frac{4900}{1960} = 2.5\,cm$$

16) (d)

$$A = (0.1)^2 = 0.01\,m^2$$

$\eta = 0.01$ poise $= 0.001$ decapoise (M.K.S unit)

$dv = 0.1\frac{m}{s}$ and $F = 0.002\,N$

$F = \eta A\frac{dv}{dx} \Rightarrow dx = \frac{\eta A dv}{F} = \frac{0.001\times0.01\times0.1}{0.002} = 0.0005\,m$ (b) $F_D = 6\pi\eta rv$

17) (b) $F_D = 6\pi\eta rv$

18) (c) If two drops of same radius r coalesce then radius of new drop is given by R

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$\frac{4}{3}\pi R^3 = \frac{4}{3}\pi r^3 + \frac{4}{3}\pi r^3 \Rightarrow R^3 = 2r^3 \Rightarrow R$
$= 2^{\frac{1}{3}} r$

If drop of radius r is falling in viscous medium then it acquire a critical velocity v and $v \propto r^2$

$$\frac{v_2}{v_1} = \left(\frac{R}{r}\right)^2 = \left(\frac{2^{\frac{1}{3}} r}{r}\right)^2 \quad \therefore v_2 = 2^{\frac{2}{3}} \times v_1$$

$$= 2^{\frac{2}{3}} \times (5) = 5 \times (4)^{\frac{1}{3}}\ m/s$$

19) (a)
20) (d)
21) (a) If velocities of water at entry and exit points are v_1 and v_2, then according to equation of continuity,

$$A_1 v_1 = A_2 v_2 \Rightarrow \frac{v_1}{v_2} = \frac{A_2}{A_1} = \left(\frac{r_2}{r_1}\right)^2 = \left(\frac{2}{3}\right)^2 = \frac{4}{9}$$

22) (a)
23) (a)
24) (a)
25) (a)
26) (c)
27) (b) According to Bernoulli's theorem
28) (a)

$$\frac{v^2}{2g} = h \Rightarrow v = \sqrt{2gh} = \sqrt{2 \times 10^3 \times 40}$$
$$= 2\sqrt{2} \times 10^2$$
$$= 282.8\ cm/s$$

29) (b)
30) (a,d)
31) (a,c,d) According to equation of continuity the volume of liquid flowing through the tube in unit time remains constant *i.e.* $A_1 v_1 = A_2 v_2$ *hence option (a) is correct According to Bernoulli's theorem*

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$$
$$\Rightarrow P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) \Rightarrow \rho g h$$
$$= \frac{1}{2}\rho(v_2^2 - v_1^2)$$

$\therefore v_2^2 - v_1^2$
$= 2gh$ *Hence option (c) is correct.*

Also, according to Bernoulli" s theorem option (d)is correct

32) (b) Horizontal range will be maximum when $h = \frac{H}{2} = \frac{90}{2} = 45\ cm\ Hole{:}3$

33) (b) Time taken to be emptied for h height.

$$t = \sqrt{\frac{2h}{g}} \text{ and for height } \frac{h}{2} \text{ height}$$

$$= \sqrt{\frac{2\frac{h}{2}}{g}} = \sqrt{\frac{h}{g}}$$

$$\therefore \frac{t'}{t} = \frac{1}{\sqrt{2}} \Rightarrow t' = \frac{t}{\sqrt{2}} = \frac{10}{\sqrt{2}} = 7 \text{ minutes}$$

34) (b)
35) (b)
36) (a)
37) (a)
38) (d)
39) (d)
40) (c) $A_1 v_1 = A_2 v_2 \Rightarrow \frac{v_2}{v_1} = \frac{A_1}{A_2} = \left(\frac{r_1}{r_2}\right)^2$

$$\Rightarrow v_2 = 3 \times (2)^2 = 12\ m/s$$

41) (a)
42) There are no viscous forces in irrotational fluid flow
43) Boundary layer thickness varies directly with the viscosity. If the viscosity of fluid is more, the thickness of boundary layer is more and vice-versa
44) The terminal velocity of sphere of radius r density ρ falling under gravity in air of viscosity of co-efficient η is given by the equation, $v_T = \frac{2\rho g r^2}{9\eta}$ We know that for water density $\rho = 10^3$ kg ms^{-3}

$$v_T = \frac{2 \times 10^3 kg^{-3} \times 9 \cdot 8ms^{-2} \times (5 \cdot 1 \times 10^{-7})^2}{9 \times 1.9 \times 10^{-5} kgm^{-1}s^{-1}}$$
$$v_T = 2.98 \times 10^{-5} ms^{-1}$$

45)

$$As\ \ d_1 = 5\ inch,\ \ d_2 = 3\ inch$$
$$v_1 = 5\frac{ft}{s}\quad v_2 = ?$$

From equation of continuity,

$$A_1 v_1 = A_2 v_2 \rightarrow \pi r_1^2 v_1 = \pi r_2^2 v_2$$
$$r_1^2 v_1 = r_2^2 v_2 \rightarrow \left(\frac{d_1}{2}\right)^2 v_1 = \left(\frac{d_2}{2}\right)^2 v_2$$
$$d_1^2 v_1 = d_2^2 v_2$$
$$v_2 = \frac{d_1^2 v_1}{d_2^2} = \left(\frac{6inch}{3inch}\right)^2 \times fts^{-1} = 7$$
$$= 20fts^{-1}$$

46) As given, D = 20mm, m = 30 kg, T = 60s, $\rho = 1000$ kg m − 3

$r = \frac{D}{2} = 10mm = 10 \times 10^{-3}m$

so $r = 0.01m$

we know that: $\frac{\Delta m}{\Delta t} = \frac{\rho \Delta v}{\Delta t} = Av$

$\Rightarrow \frac{\Delta m}{\Delta t} = Av$

$\frac{30}{60} = \pi r^2 v = (3.14)(0.01)^2 v$

$\Rightarrow 0 \cdot 5 = 3.14 \times 10^{-4} v$

$v = \frac{0 \cdot 5}{3 \cdot 14 \times 10^{-4}} = \frac{0 \cdot 5 \times 10^4}{3 \cdot 14}$

$= 1592.3ms^{-1}$

47) Option b is correct. it is in accordance with law of conservation of energy. " energy can neither be created nor it can be absorbed but it can be transformed from one form to another"

48) The relationship between pressure, flow speed and height for flow of an ideal fluid is called Bernoulli's equation. So option d is correct.

49) The unit of ρgh is same as that of pressure which is Nm^{-2} and its dimension is $ML^{-1}T^{-2}$

50) B is correct because pressure is inversely proportional to velocity(speed)

51) As we know that speed of efflux is given by the relation:

$v = \sqrt{2gh} = v = \sqrt{2(9.8)(1)} = 4.42\ m/s$

If there are two heights then formula is:

$v = \sqrt{2g(h_1 - h_2)}$

→ *it is because we always take HeigHt from top side*

52)

$v_T = \frac{2\rho gr^2}{9\eta} \Rightarrow r = 2r \Rightarrow \frac{2\rho g(2r)^2}{9\eta}$

$\Rightarrow 4\left(\frac{2\rho gr^2}{9\eta}\right) \Rightarrow 4v_T$

53) **1ST Method:** The terminal velocity of each drop is,

$v_T = \frac{mg}{6\pi\eta rv} = 6\frac{cm}{s}$

When 8 drops combine and forms single drop its mass and radius become;

$m' = 8m,\ v' = 8v \rightarrow \frac{4}{3}\pi r^3 = 8\left(\frac{4}{3}\pi r^3\right)$

$r'^3 = 8r^3,\ r' = 2r$

So the terminal velocity of the big drop is:

$v_T' = \frac{m'g}{6\pi\eta rv'} = \frac{8mg}{6\pi\eta(2r)}$

$v_T' = \frac{8}{2}\left(\frac{mg}{6\pi\eta r}\right) = 4v_T = 4 \times 6m/s$

$v_T = 24\ cm/sec$

2ND Method: Simple trick is that, Take a cube of the drops which are given in this case the given drops are 8 so the cube of 8 is 2. now the original velocity is 6cm/s, multiple cube root of the drops that are to combine with the original terminal velocity that is given, here

We know that ;

$V_T \propto r^2$

Now we know that $(2r)^2 = 4v_T$

Original velocity is given which iis 6 multiply this by 4

6 x 4=24cm/se → which is our correct

54) B is correct because speed is inversely proportional to area

55) B

56)

$v_T = \frac{2}{9} \times r^2 \frac{(\rho - \sigma)}{\eta} g$

$\frac{v_T}{r^2} = \frac{2}{9}\frac{(\rho - \sigma)g}{\eta} = constant$

→ *so option d is correct*

57) Once terminal speed is attained by the body the net force on the body is zero, therefore acceleration of the body is zero

58) According to Bernoulli's theorem at high attitude the density of air is considerably low therefore the pressure difference depend upon the attitude of air plane, so option a is correct

59)

$v_T \propto r^2 \Rightarrow \frac{v_1}{v_2} = \frac{r_1^2}{r_2^2} \Rightarrow \left(\frac{1}{2}\right)^2 = \frac{1}{4}$

→ *so 1:4 is correct*

60) From equation of continuity,

$a \times v = constant \rightarrow or\ \pi(2R)^2 \times v_A = A(r)^2 \times v_B$

$\therefore therefore\ v_B = 4v_A = 4v$

→ *so opttion d is correct*

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